A new class of focusing crystal shapes for the Bragg spectroscopy of small, point-like, x-ray sources in laser produced plasmas

M. Bitter[1], K. W. Hill,[1] Lan Gao,[1]B. Kraus, [1]P. C. Efthimion,[1]
L. Delgado-Aparicio,[1] N. Pablant,[1]
M. B. Schneider,[2]and the NIF X-ray Group[2]

[1]*Princeton Plasma Physics Laboratory, Princeton, NJ*

[2]*Lawrence Livermore National Laboratory, CA*

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I. Introduction
Challenges for the x-ray spectroscopy of HED-plasmas

The experiments at high-power laser facilities make high demands on the x-ray Bragg crystal spectroscopy with respect to spectral resolution and photon throughput.

These demands can no longer be satisfied with the presently used standard crystal forms:

- Flat crystals
- Spherically bent crystals
- Cylindrically bent crystals
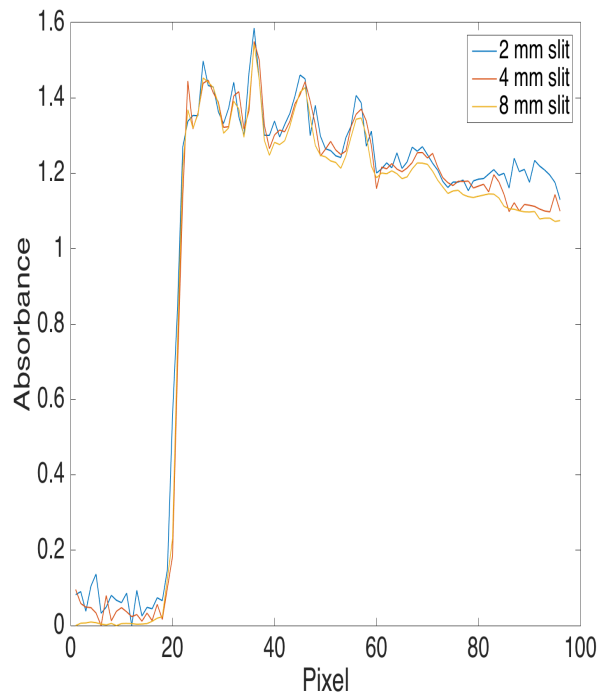- Conventional crystal toroids
- Log-spiral crystals

Therefore, entirely new focusing crystal forms are required.

These new crystal shapes must exactly satisfy the imaging conditions for each wavelength in an extended spectral range of typically 1 keV, so that - in order to enhance the photon throughput - the area of the crystal can be increased without the introduction of imaging errors.

I. Introduction
Example: *Extended X-ray Absorption Fine Structure (EXAFS)*

An example for the challenging demands are the EXAFS studies of the state of matter under extreme conditions in HED plasmas that are becoming increasingly important.

Conventional Toroid Ge220 Crystal
- Rh =889 mm, Rv = 104.8 mm requested
- 40 mm x 29 mm
- Energy range: 6.9 - 7.6 keV (W Ll line at 7.3878 keV)
- Center Bragg angle ~25 deg.
- Iron K edge at 7.112 keV

Requirements:

- High photon throughput;

- High spectral resolution of $\dfrac{E}{dE} \approx 10,000$

  for an extended spectral range of 1keV;

- Elimination of source-size broadening effects.

To satisfy this last requirement one must choose crystal forms, which have a well defined Rowland circle at each crystal point, since source-size broadening effects are minimized if the detector is positioned on a Rowland circle.

II. A new class of focusing crystal shapes for the Bragg spectroscopy of small, point-like, x-ray sources

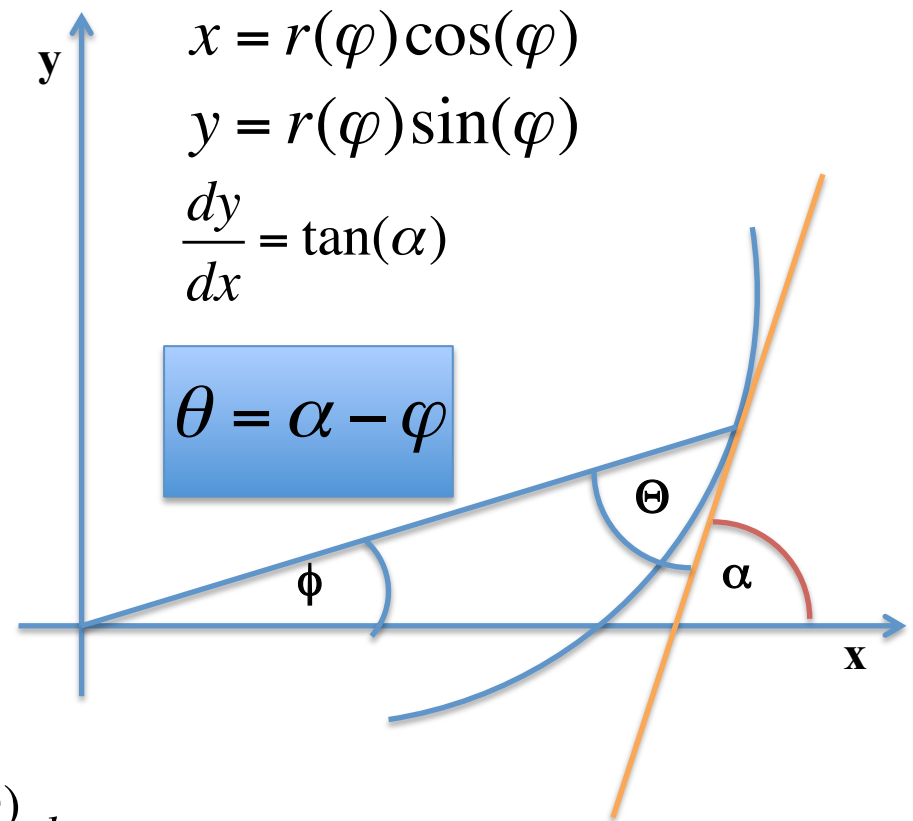- An arbitrary curve is described by the differential equation:

- (1)
$$\frac{1}{r}\frac{dr}{d\varphi} = \frac{1}{\tan(\theta)}$$

- For Θ = const., we obtain the log-spiral:

- (2)
$$r = r_0 e^{\frac{\varphi}{\tan(\theta)}}$$

- In order to find additional solutions we rewrite (1) as:

- (3)
$$\frac{dr}{r} = \frac{d\varphi}{\tan(\theta)} = \frac{\cos(\theta)}{\sin(\theta)}d\varphi = \frac{\cos(\alpha-\varphi)}{\sin(\alpha-\varphi)}d\varphi$$

$$x = r(\varphi)\cos(\varphi)$$
$$y = r(\varphi)\sin(\varphi)$$
$$\frac{dy}{dx} = \tan(\alpha)$$

$$\theta = \alpha - \varphi$$

II. Additional Solutions to the differential equation for two-dimensional curves

Making the assumption: $\alpha = \alpha_0 + b \bullet \varphi$, so that $d(\alpha - \varphi) = (b-1) \bullet d\varphi$, we obtain:

(5)
$$\frac{dr}{r} = \frac{\cos(\alpha - \varphi)}{\sin(\alpha - \varphi)} d\varphi = \frac{1}{b-1} \frac{\cos(\alpha - \varphi)}{\sin(\alpha - \varphi)} d(\alpha - \varphi)$$
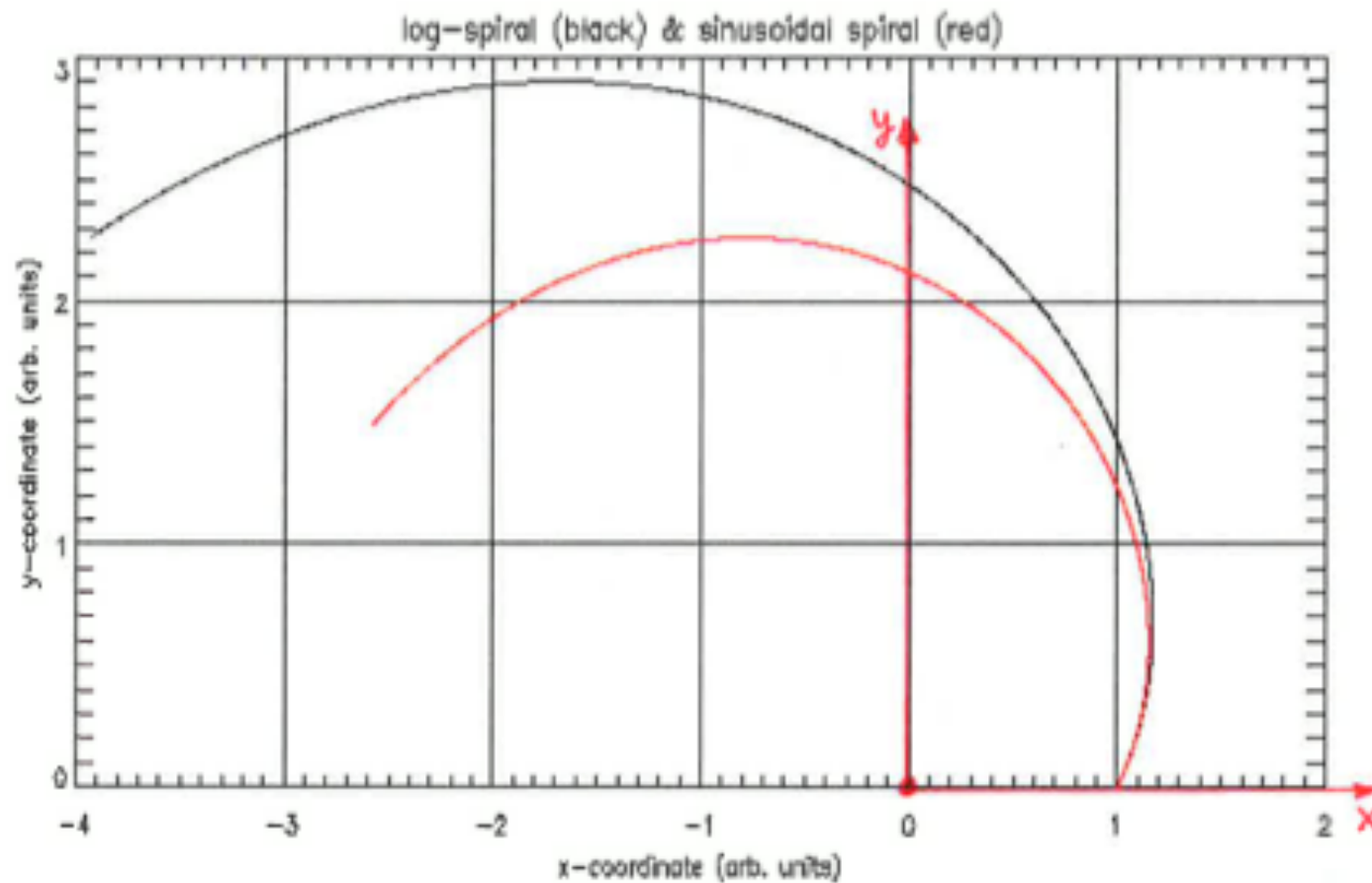
or
$$d\ln(r) = \frac{1}{b-1} d\ln\left[\sin(\alpha - \varphi)\right]$$

(6)
$$r(\varphi) = r_0 \left[\frac{\sin\left[\theta_0 + (b-1)\varphi\right]}{\sin(\theta_0)}\right]^{\frac{1}{b-1}} = r_0 \left[\frac{\sin(\theta)}{\sin(\theta_0)}\right]^{\frac{1}{b-1}}$$

The curves described by Eq. (6) are known as sinusoidal spirals. Their shape is determined by only one parameter **b** with $-\infty \leq b \leq \infty$
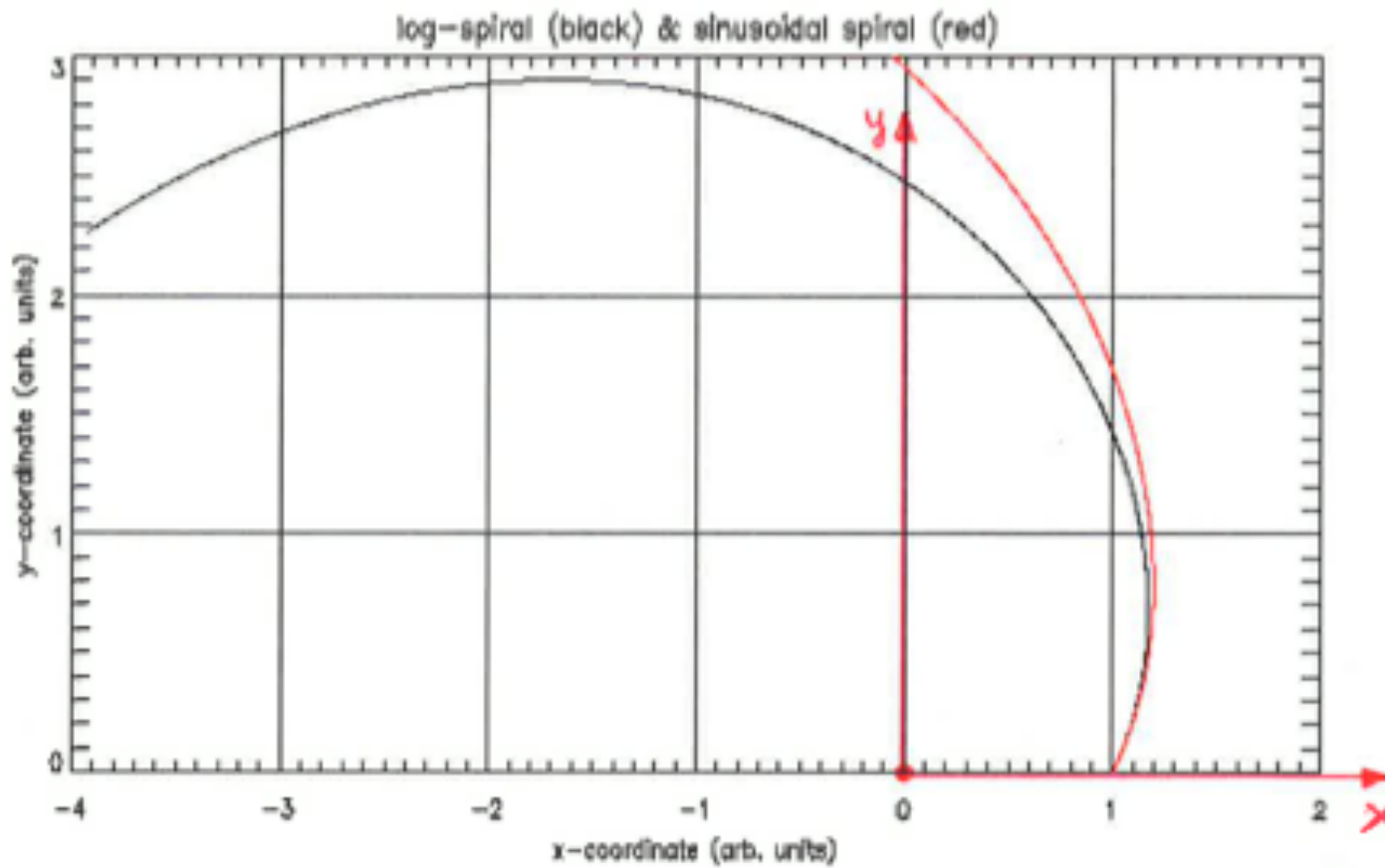
- **Spirals:**   **b = 1.1**   **THETA0 =   60°**
- -   phi(0:10) =   0.000000   0.100000   0.200000   0.300000   0.400000
- -   thetta(0:10) = 60.0000   60.0100   60.0200   60.0300   60.0400

log-spiral (black) & sinusoidal spiral (red)

- **Spirals:**   **b = 0.9**   **THETA0 =   60°**
- -   phi(0:10) =   0.000000   0.100000   0.200000   0.300000   0.400000
- -   thetta(0:10) =   60.0000   59.9900   59.9800   59.9700   59.9600

log-spiral (black) & sinusoidal spiral (red)

- **Spirals:**   **b = 1.001**   THETA0 =   60°
- -   phi(0:10) =   0.000000   0.100000   0.200000   0.300000   0.400000
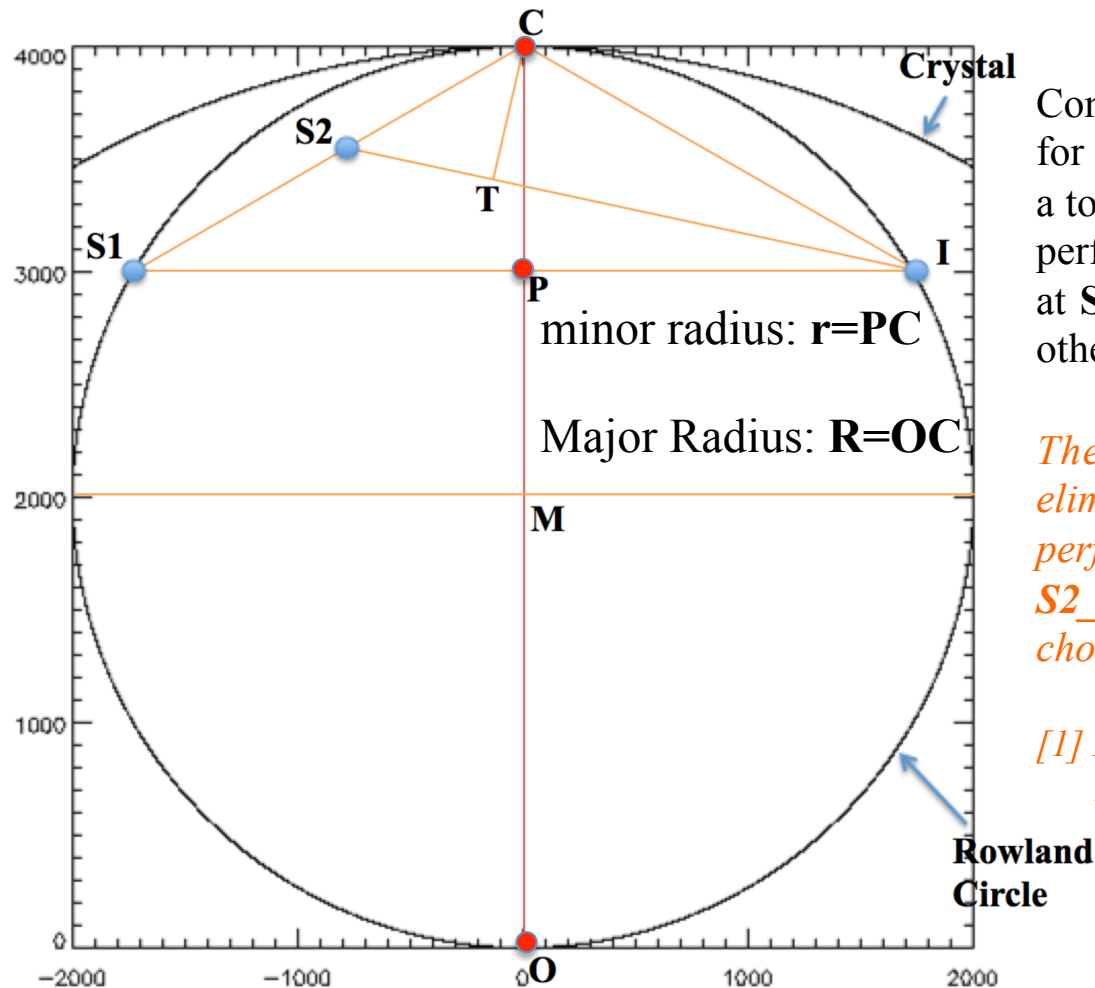- -   thetta(0:10) = 60.0000   60.0001   60.0002   60.0003   60.0004

log-spiral (black) & sinusoidal spiral (red)
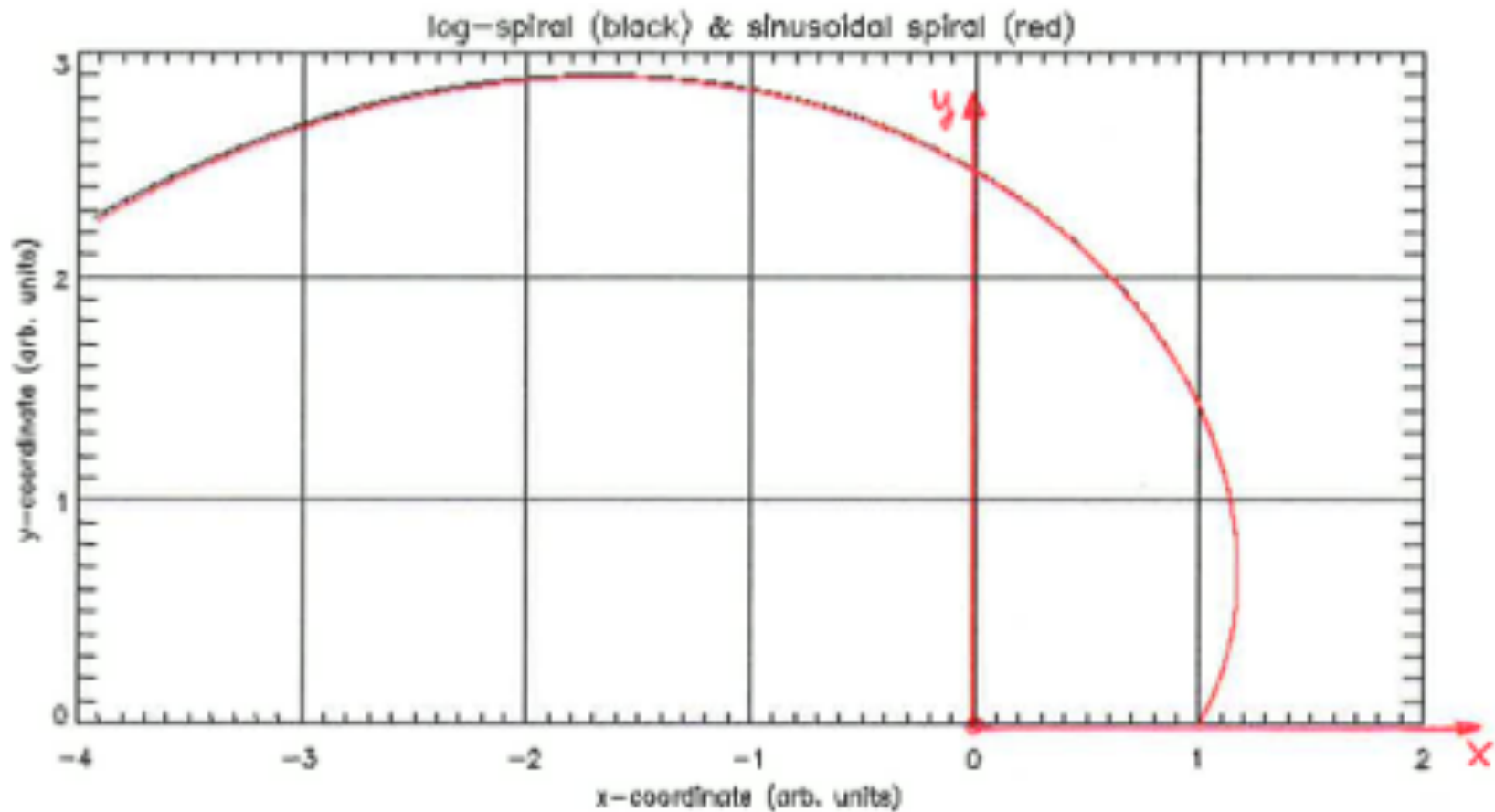
Conventional & Modified Toroid Crystal Spectrometers

(Modified Torus: major radius $R=OC$, minor radius: $r=TC$)
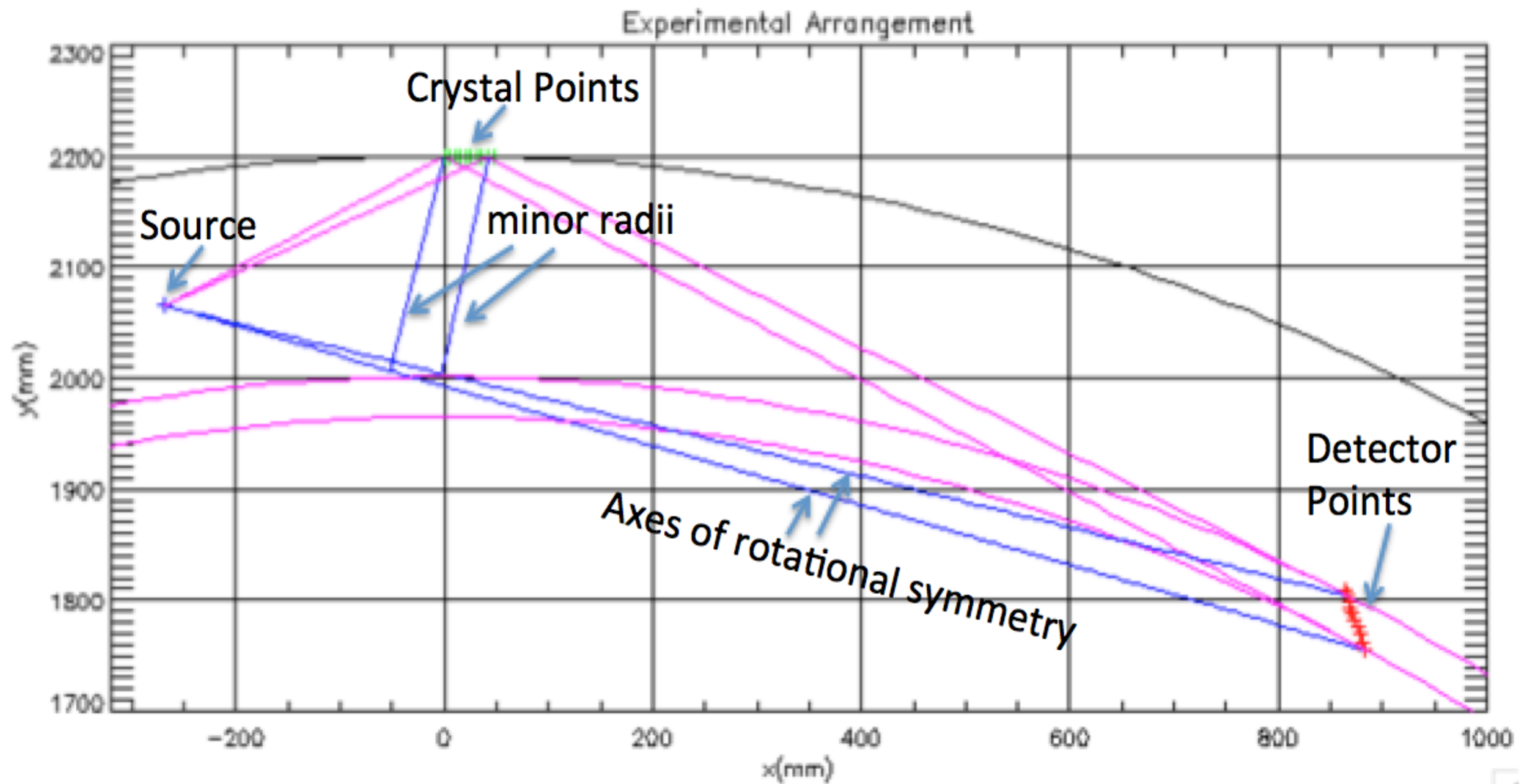

minor radius: $r=PC$

Major Radius: $R=OC$

Conventional crystal toroids are presently used for EXAFS measurements at the NIF. But, such a toroidal crystal, with a minor radius **PC**, works perfectly for only one wavelength if the source is at **S1**. For any other source position **S2** and any other wavelength one obtains imaging errors.

The modified toroidal crystal design [1] eliminates those imaging errors and provides perfect imaging for each wavelength by defining S2_I as an axis of rotational symmetry and choosing CT as the local minor radius.

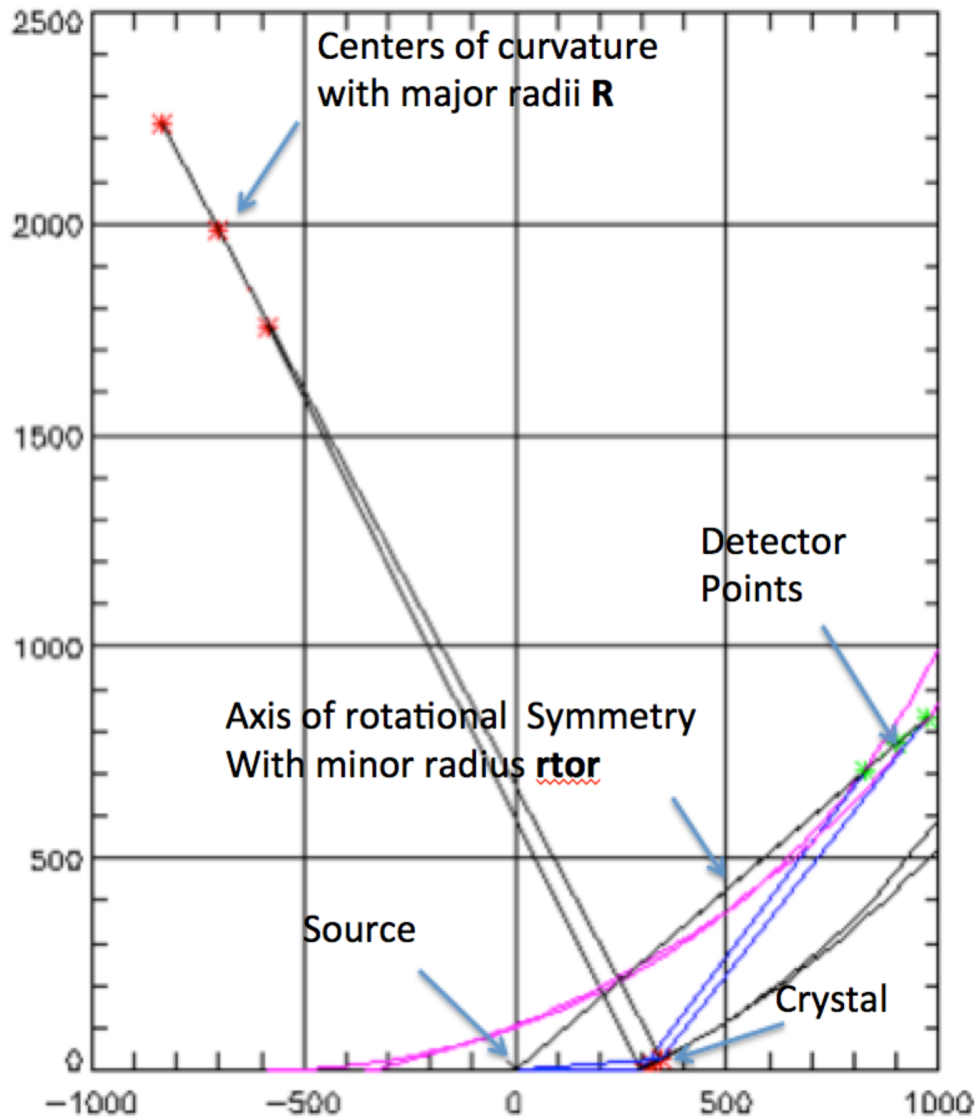*[1] M. Bitter et al., Rev. Sci. Instrum. **89**, 10F118 (2018)*

Constant major radius: R=**2200 mm**; variable minor radius **r** Source-Crystal
Distance: **Dsc = 300 mm**
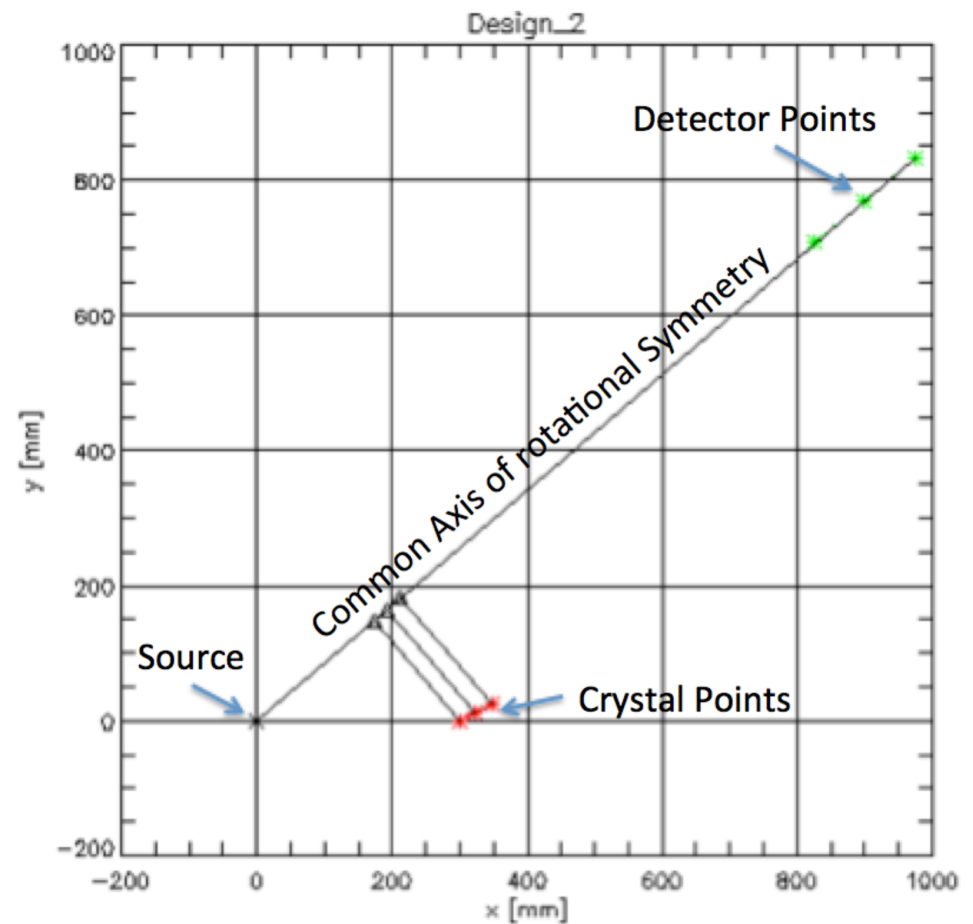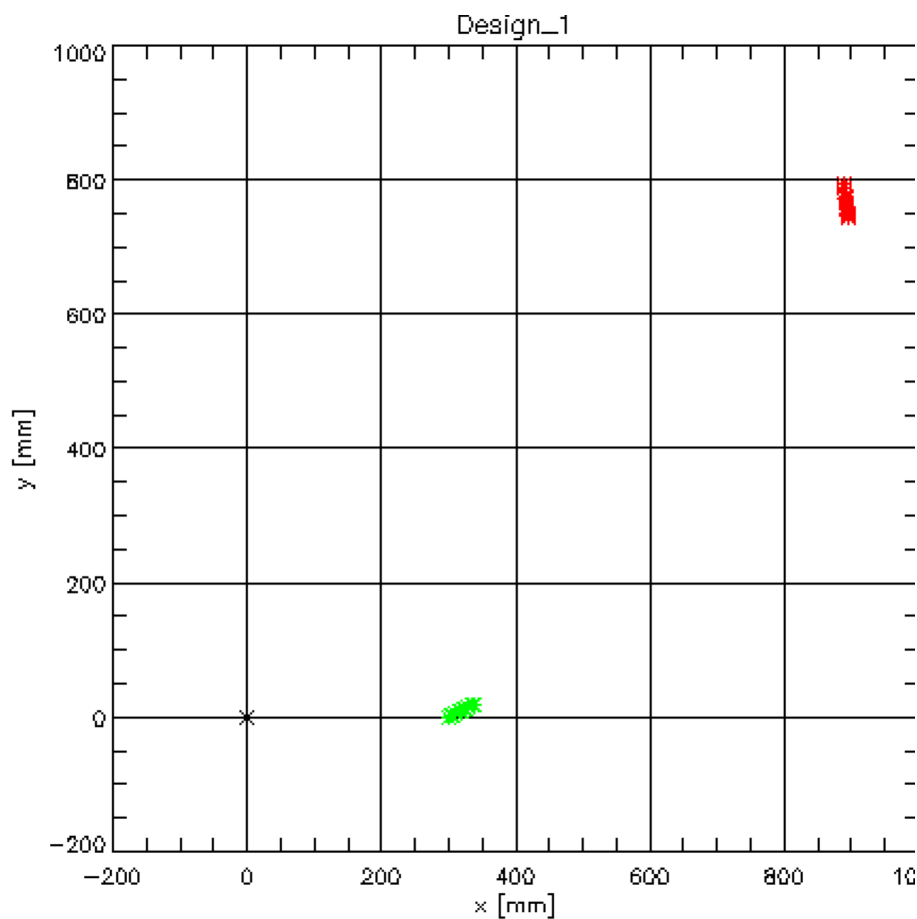
IV. *Design_2 (*Sinusoidal Sprial Toroid)

Torus for a Sinusoidal Spiral with b=0.34:

Variable major radius **R** Variable minor radius **r** Source-Crystal Distance: **Dsc = 300 mm**

*Comment: The minor radii, **r**, are perpendicular to the axis of rotational symmetry. They are therefore parallel to each other and vary only in magnitude, which is equal to the distance from the axis of rotational symmetry to the point of Bragg reflection on the crystal.*

IV. Differences between Design_1 and Design_2

Dispersion:

Design_1: $\dfrac{dE}{ds} = 15.5 [eV/mm]$

Design_2: $\dfrac{dE}{ds} = 5.1 [eV/mm]$

Spectral resolution:

(based solely on dispersion and image plate resolution of 0.1 mm)

Design_1: $\dfrac{E}{dE} = 6450$

Design_2: $\dfrac{E}{dE} = 20000$

Conclusions:
- Design_2 is superior to Design_1.
- The spectral resolution is no longer limited by the spectrometer design; it is only limited by the crystal properties.

IV. Additional Favorable Features of Design_2

- There is **only one, *i. e. a common,* axis of toroidal symmetry** for all the wavelengths, which are reflected from a different crystal points. – *This was obtained by tweaking the parameter **b**, i. e. setting **b = 0.34.***

- The minor radii **r**, which connect the crystal points with the **common axis of symmetry** are therefore **parallel** to each other; *but **r** varies in magnitude.*

- **The detector points are on a straight line,** namely, the *common* axis of rotational symmetry, since the source and all its images **must always be** on the axis of rotational symmetry.

- **Design_2 has thus much in common with the von Hamos geometry!**

- **Another important feature is that each Bragg angle interval, $\Delta\Theta$, receives the same number of photons, since $\Delta\Theta = (b-1)\,\Delta\phi$.**

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*   For questions to and further discussions of this work, please contact me per email, at: **bitter@pppl.gov.**